set to .TRUE. only if there are failures or improvements in objective function value; less than ϵ' after considering values of k_r from 1 to n_r in one complete cycle.

The variables α_o, $\breve{\alpha}$, β, ϵ, ϵ', η, ϕ^o, ψ_i, k, k_r, and n are initially assigned values on entry to the grazor search package. The subroutine ANAL(ϕ, ψ_i, DERIV, k, y_i, ∇y_i) is an analysis program to evaluate y_i and/or ∇y_i at a given point ϕ. The function subprogram $Y(\phi, \psi_i, k)$ calculates the y_i corresponding to the point ϕ by calling ANAL. The subroutine LOCATE (ϕ, ψ_i, k, n, U_ϕ) evaluates the objective function U_ϕ by calling $Y(\phi, \psi_i, k)$ for $i=1, 2, \cdots, n$.

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Anomalous Loss at a Ferrite Boundary

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Abstract—The occurrence of anomalous loss and its explanation in terms of surface waves is discussed. For this type of explanation to be possible the region of occurrence of the surface wave must at least straddle the region of anomalous loss. It is shown that this is so, particularly for the case when there is a mixed air–ferrite surface layer for which this result is not obvious: as the air content decreases, a ferrite–metal surface wave appears and takes over the function of the layer wave. The means by which these waves are generated, and the determination of their amplitudes, appear to require a new physical principle to be applied. A new type of "edge condition" is postulated.

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I. Introduction

THE FIRST intimation that something peculiar could be happening in a waveguide–ferrite configuration appeared in a paper by Lax and Button [1] and led to the so-called "thermodynamic paradox," in which energy could apparently be conveyed in only one direction in a lossless medium. Bresler [2] attempted to resolve the difficulty by a consideration of a surface-wave mode, and other explanations are also current [3]–[6]. In all these cases attention was focused primarily on the *existence* of energy-carrying modes, but without concern as to how these modes were to be excited.

In a quasi-static solution to the problem of reflection at a transversely magnetized ferrite block in a rectangular waveguide, Lewin [7] pointed out that there was an anomalous region in which the discontinuity admittance contained a substantial resistive part despite the assumed absence of material losses. The anomalous region occurs when the internal biasing field H_0, the angular frequency ω, and the saturation magnetization M_0 are related by the inequality $\gamma|H_0+M_0/2| <\omega<\gamma|H_0+M_0|$, where γ (positive) is the gyromagnetic ratio. Here it is convenient to define $\omega_H=\gamma|H_0|$ and $\omega_M=\gamma|M_0|$. The sign of H_0 and M_0 is, in a sense, arbitrary, and determines whether the energy absorption effect occurs at the wall $x=0$ or $x=a$. It appears in the form of a "hot line" at the intersection of the ferrite–air interface and the guide wall.

The reality of, and explanation for, this anomalous loss has been the subject of a number of papers [8]–[15], whose essential conclusions are discussed later. Although the phenomenon has not, apparently, been put to any practical use, it is not unimportant for the following reasons.

1) It could interfere with the correct operation of a microwave device whose design did not adequately take into account its possible occurrence.

2) Since it apparently leads to the generation of very large microwave magnetic fields, it could cause the appearance locally of nonlinear phenomena in situations in which only linear (small-signal) effects are expected.

3) The manner of operation of the uniqueness theorem in electromagnetic theory, and the possibility of expansion of waveguide fields as a sum of modes under all possible circumstances, is questioned from an unusual point of view.

4) The possibility of "intrinsic loss" due to infinite energy concentration in nominally lossless materials has been suggested as a loss mechanism, and the validity of this thesis requires clarification.

5) The operation of a hitherto unsuspected constraint on the order of infinity which can be admitted near a field singularity is involved and the validity and cause of this limitation requires elucidation.

Of course, all materials are lossy, even if only to a small extent, so considerations of purely lossless conditions may seem academic. But usually as the loss parameter goes to zero so do the actual losses. This is not so in the case of a ferrite working in the anomalous region, and a certain amount of closely argued analysis, in places bordering rather dangerously on hair splitting, is necessary to disentangle the different factors at work. It will be considered here that "lossless" means "in the limit of zero loss parameter," the essential feature being the limiting process. The only attempt to deal with a strictly zero loss problem [11] was not successful, and no meaning is given here to zero loss in this strict sense.

Although the reality of the anomalous loss phenomenon has been vividly demonstrated [12], [15], and is not in dispute, the causative mechanism has been variously allocated to crack waves [8], metal-boundary surface waves [3], [9], and intrinsic loss [4]. In a practical situation all three effects are probably present and are presumably all aspects of one and the same phenomenon, which seems related to a high concentration of microwave energy near the ferrite boundary. Hurd, in a recent paper [14], has endeavored to make a case for the intrinsic loss mechanism only, to the exclusion of the surface-wave or crack-wave explanations. This claim is examined here, and it is shown that his results are quite compatible with the existence of these waves. The matter is not a purely academic one, since the strength of the microwave magnetic fields at the ferrite boundary is different in the two cases.

II. Discussion of the Previous Results

Lewin [8] accepted the reality of the anomalous loss effect, and attempted to explain it as due to Bresler's [2] "crack wave," a wave shown to occur in the minute crack assumed to exist between the ferrite and waveguide wall. This explanation was not very convincing because the original analysis had ignored any such wave. The analysis of the boundary field as an infinite sum of waveguide modes, normally considered adequate in these problems, becomes suspect if a crack wave must also be considered. The usual mode expansion is contingent on the conditions for validity of a Fourier sine series expansion, and this includes a requirement of limited fluctuation [16]. In the present instance, in the absence of the crack wave, the magnetic field near the wall $x=a$ varies as $(a-x)^{1-2p}$, where p is a complex parameter with $0<\operatorname{Re} p<1$. The fluctuations are like the sine and cosine of $[2 \log (a-x) \operatorname{Im} p]$ and have zero period as $x \to a$. So a reason for something unusual happening at $x=a$ certainly exists; and Bresler's crack wave seemed just what was needed, since its possible occurrence was in *exactly* the same range of parameters as that of the anomalous loss. Perhaps too much was made of this coincidence in the earlier papers. What is essential for such an explanation, however, is that the region of existence of the crack wave should at least *straddle* that of the anomalous loss; it should always be *possible* for the wave to be there when needed. (It is perhaps unimportant if the wave *could* exist outside the region where it is really wanted to provide the explanation of the loss mechanism.)

Barzilai and Gerosa [3] considered an alternative cause of loss, a surface wave at the ferrite–metal boundary at the waveguide wall. This wave is very lossy and has a complex propagation coefficient

$$\gamma = (\kappa - \mu)\left(\frac{j\omega\mu_0\sigma}{\mu_0^2 - (\mu - \kappa)^2}\right)^{1/2} \tag{1}$$

where μ and κ are the elements of the ferrite permeability tensor and σ is the metal conductivity. The wave exists as an actual electromagnetic entity provided the attenuation is positive, or $\operatorname{Re}\gamma > 0$. This requires $\kappa > \mu$ and also $\mu_0^2 > (\mu - \kappa)^2$. Since $(\mu - \kappa)/\mu_0 = 1 + \omega_M/(\omega_H - \omega + j\alpha)$, where ω_M, ω_H, and α, the loss parameter, depend on the ferrite properties, these conditions give $(\omega_H + \omega_M/2) < \omega < (\omega_H + \omega_M)$ as $\alpha \to 0$. This is just the anomalous region, so that the existence of the boundary wave is guaranteed when it is needed to explain the loss mechanism.

In a subsequent paper [9] Gerosa proceeded to analyze the ferrite configuration, taking into account this surface wave. Various approximations were made, consistent with the metal conductivity being large, and an integral equation was eventually obtained identical with Lewin's original one except for the addition of a surface-wave term.

This analysis is noteworthy for two reasons. It was the first which sought to take account of a surface wave *ab initio*. And it contained an *arbitrary* amplitude of the surface wave. An attempt was made to determine this amplitude from orthogonality considerations, but this turned out to be unsatisfactory, and at this stage it subsequently became clear that the problem was in some sense inadequately defined. This was brought out fully in a later paper by Lewin [10], who showed that *all* the usual boundary conditions were met, *even in the absence of the surface wave*. And, in fact, since it was not needed under those conditions when, in any case, it could not exist, the solution obtained in that range of parameters, the normal or "nonanomalous" range, *must continue* to satisfy Maxwell's equations *and* the boundary conditions also in the anomalous range, since the geometry had not changed merely by virtue of changing, say, the ferrite magnetization. The peculiar situation was reached that a solution had been obtained, which according to electromagnetic theory must be a unique one, and yet in the anomalous region an apparently arbitrary amount of surface wave could be added. Either this surface wave was a spurious entity, or else some other rather recondite condition must determine its amplitude.

There seems to be no escape from this conclusion. Lewin [10] postulated a new condition, somewhat analogous to an edge condition: A restraint on the possible order of infinity of the magnetic field at the edge of the boundary. This enabled a unique solution to be found. It needs to be stressed that, without this condition, or some other similar principle, the solution contains an arbitrary undetermined constant. To *ignore* the surface wave (by taking its amplitude as zero) is, of course, equally arbitrary, and whether or not it is the cause of the anomalous loss, *something* has to determine its amplitude. This something is *necessarily new*, since all the usual boundary conditions, field matching, etc., have already been used in setting up the problem. Lewin's postulate, if correct, ought to be obtainable from some physical principle, such as finiteness of stored energy, or the like. So far this has not been demonstrated, and its validity must be considered tentative.

An attempt to bypass all these troubles was made in a paper by Mittra and Lee [11] in which the ferrite loss was taken to be accurately zero from the start. Their solution, however, is faulty [5], [13] in that it fails to give zero tangential electric field at the guide walls, as well as implying a nonphysical negative loss if the ferrite is slightly lossy.

Hurd [6] opts for a concept labeled "intrinsic loss." Essentially, if the ferrite is lossy, with loss parameter α, then, as α goes to zero, the permeability tensor can be put in the form $[\mu]_{\mathrm{H}} + \alpha[\mu]_{\mathrm{NH}}$, where H and NH stand for Hermitian and non-Hermitian, respectively. The first term gives no loss, and the second term could do so if the magnetic field became infinite as α went to zero in such a way that a finite product resulted. Hurd [4] demonstrates this possibility, but clearly the conclusion is affected if a surface wave can be present of such form and magnitude as to *cancel* the dominant infinity of the magnetic field. This is just the effect of Lewin's postulate, and although its validity has not yet been demonstrated, it is worth reiterating that *some* such principle is needed to obtain a unique solution to the problem.

III. A Ferrite-Layer Wave

Hurd [14] comments, and validly so, that to assume an air crack between ferrite and guide wall is too arbitrary, and that if an imperfection of this sort is to be examined, the parameters of the crack infilling should more realistically involve the constants of the ferrite, air, and metal boundary. He actually examines a ferrite infilling of arbitrary parameters intermediate between air and the main ferrite and shows that it can support a surface wave (in fact *three* surface waves) which reduces to Bresler's crack wave for a wholly air boundary. But in general the conditions for existence of this wave differ noticeably from the conditions for existence of the anomalous region. From this, Hurd concludes that the association of the surface wave with the anomalous loss is rendered rather unlikely.

In fact the association would be *impossible* unless there were a sufficient overlap—the anomalous region, if not coinciding with the surface-wave region, *must* be inside it. We proceed to show, with an appropriate proviso, that this is indeed so, thus rendering permissible the surface-wave interpretation.

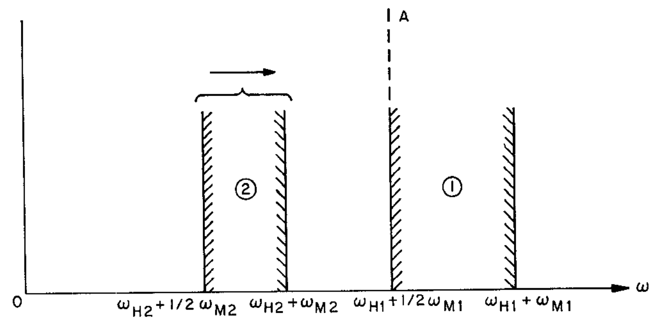

Fig. 1. Location of anomalous regions for main ferrite ① and surface layer ②. The arrow indicates the movement of the anomalous region of the surface layer as its ferrite content (and hence magnetization) is increased. The position marked A represents the location of the value of ω for the lower limit of anomalous loss in the main ferrite.

Let us look a little more closely at this "crack" layer from two extreme points of view: Near-air composition, and near-main-ferrite composition. We suppose the frequency to lie in the anomalous range $(\omega_{H1}+\omega_{M1}/2)<\omega<(\omega_{H1}+\omega_{M1})$ for the main ferrite, using subscripts 1 for this ferrite and 2 for the crack layer. Then for near-air composition the condition of the layer would correspond to very weak magnetization, with $\omega\gg(\omega_{H2}+\omega_{M2})$, so that the layer is in the nonanomalous condition. Under these circumstances Hurd's equations (8) determine a surface wave in the region $r_2<\omega<(\omega_{H1}+\omega_{M1})$ where r_2 (the negative of Hurd's r_2) can be readily shown to be less than $(\omega_{H1}+_{M1}/2)$. Hence this range indeed straddles the anomalous range, and it does so until, with increasing magnetization, r_2 has to be replaced by $(\omega_{H2}+\omega_{M2})$. This occurs at a value approximately given by $\omega_{M2}=\omega_{M1}/3$, after which this lower limit, as the magnetization is further increased, approaches $(\omega_{H1}+\omega_{M1}/2)$. At this point, as indicated in Fig. 1, the crack layer *itself enters an anomalous condition*, and the bare configuration investigated by Hurd can no longer be considered an appropriate representation of the ferrite-filled guide arrangement. It would be necessary, for instance, to treat the thin ferrite layer adjacent to the guide wall in the way pursued by Gerosa [9], taking into account the finite conductivity of the metal wall. If this is done, then an additional surface wave is introduced, and this fulfills the function of the now no-longer-existing crack wave. As the one goes out the other comes in! The fact that the crack layer is thin is irrelevant. It is in an anomalous condition and, being adjacent to the metal wall, *must* support a surface wave there.

We conclude that Hurd's analysis, far from undermining the possible role of the surface or crack waves, goes a long way in support in so far as it discloses that these waves are *always there when needed*—a *sine qua non* for a nonintrinsic loss explanation.

IV. Conclusions

The role, if any, of surface or crack waves in the absorption of energy in the anomalous region remains undetermined, though nothing in recent analyses in any way refutes the idea of their constituting the basic mechanism. However, *irrespective of their role*, the question of the determination of their amplitudes poses a radically new theoretical problem. Lewin's postulate may or may not survive a more thorough investigation, but *some* such new principle would appear to be quite essential to resolve this problem.

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